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Research Article

Chaotic dynamics in Turkish foreign exchange markets

Türkiye Forex piyasalarının kaotik dinamikleri

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Abstract

The financial systems, and particularly foreign exchange markets, are complex. This study investigates whether the Turkish foreign exchange market exhibits chaotic dynamics. To achieve this goal, we focus on a currency basket composed of equally-weighted Euro and US Dollar against the Turkish Lira. We compute Lyapunov exponents (LE) embedded in daily data of the currency basket from 2018M05D01 to 2022M05D23. The time interval under examination also indicates Turkey's alternative economic and financial policies have been preferred. We employed the phase space reconstruction method, which enables the detection of multiple equilibria in foreign exchange markets due to recent monetary policy applications. The study's main findings demonstrate that the daily currency basket data exhibit chaotic behaviour, and the associated maximal Lyapunov exponent is positive. An increase in complexity may recursively cause volatility in exchange rates for some time interval. Therefore, in policy-making, finding the root factors that sustain the chaotic behaviour of exchange rates and preventing multiple equilibria on expectations is crucial, which leads back to volatility. The study findings have important implications for interventions of Central banks as well as portfolio and risk management.

Keywords: Chaos, Lyapunov Exponent, Efficient Market Hypothesis, Exchange Rate, Turkish Lira

Jel Codes: C32, G14, G17

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Öz

Finansal sistemler, özellikle de kur piyasaları kompleks sistemlerdir. Bu çalışmamızda Türkiye kur piyasalarının kaotik dinamik sergileyip sergilemediğini araştırıyoruz. Bunun için, Euro ve ABD Doları'nı eşit ağırlıklı alarak meydana getirdiğimiz kur sepetini inceliyoruz. Bu kur sepeti bir finansal zaman serisi teşkil eder, bu seriyi 2 Mayıs 2018 tarihinden 23 Mayıs 2022 tarihine kadar günlük gözlemlerle oluşturduk ve bu serinin Lyapunov katsayılarını hesapladık. Araştırmamıza konu olan zaman dilimi, Türkiye'de alışılmışın dışında, alternatif ekonomi politikalarının uygulandığı bir dönemdir. Faz-uzayı yapılandırması metodunu kullandık para politikası uygulamalarının sonucu olarak ortaya çıkan çoklu-dengeleri araştırdık. Çalışmamız göstermektedir ki, kur piyasaları kaotik bir davranış sergilemektedir, sepet kur serisi kaotiktir ve pozitif Lyapunov katsayısı hesaplanmıştır. Piyasalarda karmaşıklığın artışı kurlarda geri beslemeli volatilite artışına sebep olabilir ve süregen volatilite artışları politika yapıcılar açısından sorun teşkil eder. Eğer kaotik davranışın nedenleri kısa-dönemde bulunmaz ve çoklu-dengeleri ortadan kaldıracak politikalar, sözlü yönlendirmeler ortaya konmazsa, beklentiler bozulur. Bulgularımızın Merkez bankası müdahaleleri, portföy ve risk yönetimi açısından önemli sonuçları vardır.

Anahtar Kelimeler: Kaos, Lyapunov Katsayısı, Verimli Piyasalar Hipotezi, Sepet Kur, Türk Lirası

JEL Kodları: C32, G14, G17

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Introduction

The exchange rate dynamics influence financial stability, which has been targeted as a policy implication simultaneous to macroeconomic stability (Calvo & Reinhart,2002). Especially in emerging markets, the exchange rate movements affect the sustainability of many other macroeconomic parameters, such as interest rates, trade balance, output and asset prices (Obstfeld, Shambaugh & Taylor, 2005). More specifically, currency markets may exhibit non-linear dynamics with chaotic behaviour. In policy-making, finding the root factors that sustain the chaotic behaviour of exchange rates and preventing multiple equilibria before volatility occurs is crucial. Otherwise, long-term macroeconomic stability will be violated. This study investigates Turkey's chaotic behaviour in foreign exchange markets based on the most recent observations.

Foreign exchange and other financial markets have long been assumed to be efficient from the dominant theoretical perspective of the Efficient market hypothesis (Fama, 1970). However, in the '90s, the efficient market hypothesis (EMH) was under severe siege. On the one hand, various papers suggested that standard risk measures do not fully explain financial markets. On the other hand, the rapidly growing literature on non-linear dynamical analysis enabled researchers to establish a different type of model specifications and to test the validity of EMH concerning complexity and non-linearity (Scheinkman & LeBaron,1989; Abhyankar, Copeland & Wong 1997; Hagtvedt, 2009; Ozkaya, 2015). It has been widely accepted that financial systems have been complex for two decades. As global financial integration broadens among multiple decision-makers and counterparties, the complex dynamics of the financial systems strengthen and enlarge. Therefore, complex financial systems can exhibit the following dynamic features: non-linearity, path dependency and sensitive dependence to initial conditions. It is a well-known fact that all these features are related to the chaotic dynamics of markets and can occur mainly in stressful times. The importance of the topic of our study can be outlined in two dimensions. From the theoretical perspective of the Efficient market hypothesis, the market prices fully reflect every piece of information which decision-makers have homogenously, instantaneously and rationally processed. Thus, the term efficiency is assimilated to linear stochastic processes, specifically random walk. As a result of this hypothesis, the market prices would be unpredictable, and financial actors cannot enjoy any arbitrage gain. Hence can obtain returns directly proportional to the risk they have taken (Malkiel, 2003). Otherwise, if the market behaviour becomes chaotic, EMH will not be realized, the market equilibrium will be disrupted, and arbitrage will occur (Brock, Lakonishok & LeBaron, 1992; Gencay, 1998). An increase in complexity decreases the available information and leads to augmented uncertainty in the foreign exchange market (Ehrmann, Fratscher & Rigobon, 2011). This causes volatility in exchange rates. Because of the volatility spillover, stock markets, bond markets and other financial markets face with spillover effect (Rigobon & Sacks, 2003; Badshah, Frijns, & Tourani-Rad, 2013). In case of insufficient foreign reserves, the foreign exchange market can lose its depth in the transaction volume, and predictability occurs (Jegadeesh, 1990). Beginning in 2018, this has been observed in Turkey, disrupting the country's financial stability. Based on macroprudential tools, it becomes increasingly difficult for the Central bank to sustain the macroeconomic stability in case of multiple equilibria in the inflation and interest rates process (Calvo, 1988). The decision-makers demand more foreign reserve currency, and the dollarization overwhelms them.

This study aims to identify the multiple equilibria in foreign exchange markets and determine the Turkish Lira's chaotic behaviour against Euro and USD Dollar from 02/05/2018 to 23/05/2022. To the best of knowledge, this is the first study investigating the chaotic behaviour of the Turkish Lira against the Euro and USD Dollar. In our study, we analyse whether the currency basket composed of Euro and USD against the Turkish Lira exhibits chaotic behaviour or not. The data covers daily observations over the period from 02/05/2018 to 23/05/2022. Therefore, this study does not need Ethics Committee Approval for the data. First, we reconstruct phase space to observe multiple equilibria embedded in financial time series, and then we compute the Lyapunov exponents of the associated dynamical system.

For this reason, this study contributes substantially to the literature from various aspects. First, in the case where market efficiency disappears, predictability and excess return possibility occur. Policymakers must monitor this situation in foreign exchange markets and impose macroprudential measures to increase the financial system's resilience to shocks by addressing possible systemic risks. Second, our study determines the effectiveness of the Central bank policy. Under the chaotic behaviour of the currency markets, Central bank intervention in foreign exchange markets will not give the desired results. Another contribution is that to sustain long-term macroeconomic stability, and policymakers should understand and prevent multiple equilibria in the financial system before chaotic behaviour occurs and expectations diverge.

The study path can be summarized as follows: following the introduction, a second part is a literature review with theoretical and empirical studies that shed light on the linkage between theory and application. In this part, we present a literature review, the methodology and a brief mathematical definition of the implemented algorithm. The third part introduces the background information on research and methodology. After presenting the data and the result of the chaotic behaviour analysis of the exchange rate, we provide discussions and implications. Finally, this paper concludes with key points, recommendations, future research directions and limitations.

Literature review

This section presents a brief and compact review of the literature focusing on non-linearity, complexity and chaos in financial markets. Table 1 tabulates a summary of the literature. These studies are also critical in another respect and lead to further progress on predictability analysis, which is crucial in portfolio management and policy making. Since this paper is the first definitive and empirical examination of chaotic behaviour in Turkish foreign exchange markets, we find it helpful to present similar studies employed for other countries and financial markets. Even though the methods in the literature differ from each other in various respects, the fundamental rationale behind them is familiar: the patterns in market prices are assumed to recur in the future, and thus, these patterns violate the market efficiency (Lehmann,1990; Edgar,1991; Vaidyanathan & Krehbiel 1992; Peters, 1991; Gencay, 1998; Das & Das 2007; Ozkaya 2015).

Theoretical and conceptual background

The currency markets may exhibit non-linear dynamics with chaotic behaviour. From the perspective of policy making, it is crucial to find the root factors that sustain the chaotic behaviour of exchange rates and prevent multiple equilibria before volatility occurs. Otherwise, long-term macroeconomic stability will be violated. The second aim of introducing Table 1 is to underline the methods used in the literature. First, we must note that various methods exist to compute Lyapunov exponents embedded in a given time series. These methods were initialized by the pioneering study of Wolf, Swift, Swinney & Vastano (1985) and developed mainly in the mid-'90s. In our study, we employ the methodology introduced by Kantz (1994). On the other hand, concerning the method of Wolf et al. (1985). Kantz's algorithm does not use the precise value of the embedding dimension. Furthermore, Kantz's approach allows us to assume noise in data which may reach an amplitude in large ranges and well-operate even with short data sets. We introduce details in the next section. Another point to be mentioned is that in the case of the Lyapunov, exponents are optimistic. Then the system is accepted to be non-linear (Grassberger & Procaccia, 1983; Eckmann, Kamphorst, Ruelle & Scheinkman 1988; Gencay & Dechert 1992). In Table 1, we tabulate the studies reported in the literature. The studies that focus on computing Lyapunov exponents LE are identified by the indication in the column Method.

Empirical review and hypothesis development

Table 1: Summary of Literature Review

Authors	Subject	Sample Info	Method	Findings
Vasilios,Rangan,Gil-Alana & Wohar (2019)	BRICS countries exchange rates	1812-2017	Lyapunov exponent	Chaotic
Eldridge & Coleman (1993)	FTSE-100	1984-1987	Lyapunov exponent	Chaotic
Hsieh (1993)	Foreign currency	1985-1990	Tests of linear and nonlinear predictabilities	Chaotic
Quang (2005)	Czech stock index PX50	1997-2005	Lyapunov exponent	Chaotic
Vassilicos, Demos and Tata (1992)	NYSE		Lyapunov exponent	Random process
Kodres & Papell (1991)	Swiss Franc, British P, JYEN, Canadian D, DM,	1973-1987	BDS test	Nonlinear
Panas & Ninni (2000)	Petroleum markets	1994-1998	Lyapunov exponent	Chaotic
Serletis & Dormaar (1996)	US Dollar and Australlian Dollar	1987-1993	Lyapunov exponent	Chaotic
Peters (1991)	S&P 500 index	not given	Lyapunov exponent	Chaotic
Das & Das (2007)	Foreign Exchange Rates	1971-2005	Lyapunov exponent	Chaotic
Tiwari et al. (2019)	Stock markets in G-7 countries	1884-2010	Lyapunov exponent	Evidence of chaos for all stocks.
Mishra, Sehgal, Bhanumurthy (2011)	Indian stock market	1991-2010	Lyapunov exponent	Evidence of chaos

Source: Author

Research and methodology

Let us denote the dynamical system, $h: R^n \to R^n$, with the trajectory,

$$y_{t+1} = h(y_t) + u_{t+1}\ ,\ t = 0,1,2 \dots, T \tag{1}$$

The dynamical system may be supposed to convey noise. The Lyapunov exponents for such a dynamical system are measures of the average rate of divergence or convergence of a typical trajectory. The trajectory can be rewritten in terms of the iterates of the function $h(.)$. For an n-dimensional system, there are n exponents which are customarily ranked from largest to smallest as given in Eq.(2):

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \tag{2}$$

Most generally, we cannot observe the states of the system and the actual functional form that generates the dynamics (Ozkaya, 2015). The model that is widely used is the following: associated with the dynamical system in Eq. (1), there is a measurement function $f: R^n \to R$ which generates the time series,

$$\varphi_t = f(y_t) \tag{3}$$

We assumed that the sequence gives the observed data $\{\varphi_t\}$. Let the target system be dynamic. This system is defined in Eq. (1). Moreover, let the observed financial time series be the output of a measurement function. This function is defined in Eq. (3). Accordingly, Takens's theorem (Takens, 1981) indicates that the reconstructed trajectory is an embedding of the original trajectory as long as the embedding dimension is sufficiently large. The method of embedding space needs two parameters, embedding dimension, m and time delay, d respectively. Estimating the time delay determines the frequency value equal to the dominant power spectral feature. Based on the observed time series $\{\varphi_t\}$, the data vector given in (4) can be generated.

$$z_t = \left(\varphi_{t+(m-1)d}, \varphi_{t+(m-1)d}, \dots, \varphi_t\right) \tag{4}$$

This vector depicts a point of m-dimensional reconstructed phase space R^m. In this setting, m can be computed by the method proposed in Abarbanel (1995). A trajectory is then drawn in the m-dimensional phase space concerning t. In this study, time delay d, is equal to 1. The reason is that our observation interval in the time domain is one.

To further progress, we define the distance between a reference trajectory z_t and one of $\varepsilon-$neighbour (s) $z_{t+\tau}^{(\varepsilon)}$ after the iteration through τ by a function $G(.): R^m \to R$

$$G\left(z_t, z_t^{(\varepsilon)}; \tau\right) = \left|z_{t+\tau} - z_{t+\tau}^{(\varepsilon)}\right| \tag{5}$$

Eq. (5) defined the vector measuring the differences between $\left(z_{t+\tau} - z_{t+\tau}^{(\varepsilon)}\right)$. In order to compute the maximal Lyapunov exponent, first, fix t, and determine the neighbours $z_t^{(\varepsilon)}$of z_t, within range of a predetermined $\varepsilon-$neighbourhood and compute the average of the distances among neighbouring trajectories. We use the logarithm of $G(.)$ in (6) since we make the output of the summation function smoother. These algorithmic steps lead us to Eq. (6). Equations (4), (5) and (6) respectively constitute the algorithm proposed in Kantz (1994).

$$D(\tau) = \frac{1}{T}\sum_{t=1}^{T} ln\left(\frac{1}{|U_t|}\sum_{z_t^{(\varepsilon)} \epsilon U_t} G\left(z_t, z_t^{(\varepsilon)}; \tau\right)\right) \tag{6}$$

In Eq. (6) $|U_t|$ denotes the cardinality of the set of $\varepsilon-$neighbours of z_t. The first sum in the RHS in Eq. (6) is composed of the distance vector among z_t and its neighbourhood. The final step is to obtain the average magnitude of the principal axis of the m-dimensional ellipsoid. Computing the slope of the curve $D(\tau)$ yields the maximal Lyapunov exponent:

$$\frac{\partial D(\tau)}{\partial \tau} \cong \lambda_{max}(t) \tag{7}$$

In Eq.(7), we obtain the maximal Lyapunov exponent defined by Kantz (1994). In summary, our numerical value for the maximal Lyapunov exponent is the slope of the curve $D(\tau)$ in the scaling region, $\tau \leq \tau_{max}$.

Findings and results

The observed time series $\{\varphi_t\}$ is a daily currency basket composed of $0.5 * $ Euro and $0.5 * $ US Dollar from 02/05/2018 to 23/05/2022. Central Bank of Turkey closing buying prices is used. The Tisean Package introduced by Hegger et al. (1999) is used to obtain Eq. (7) results. All computations, Figure 1 and Figure 2 are obtained using the R programme. In Figure 1, we depict the scatter plot visualization of the exchange rate series. The z-axis denotes the time series, the y-axis denotes the series 1-lagged behind, and the x-axis denotes the 2-lag behind. This is a phase-space representation of exchange rate series in a 3-dimensional phase-space. Figure 1 shows the series as a line graph. This representation enables us to reveal recurrent patterns. On the other hand, we present Figure 2 to detect multiple equilibria. Figure 2 is obtained from Eq. (4) and is the phase-space reconstruction of the currency basket series.

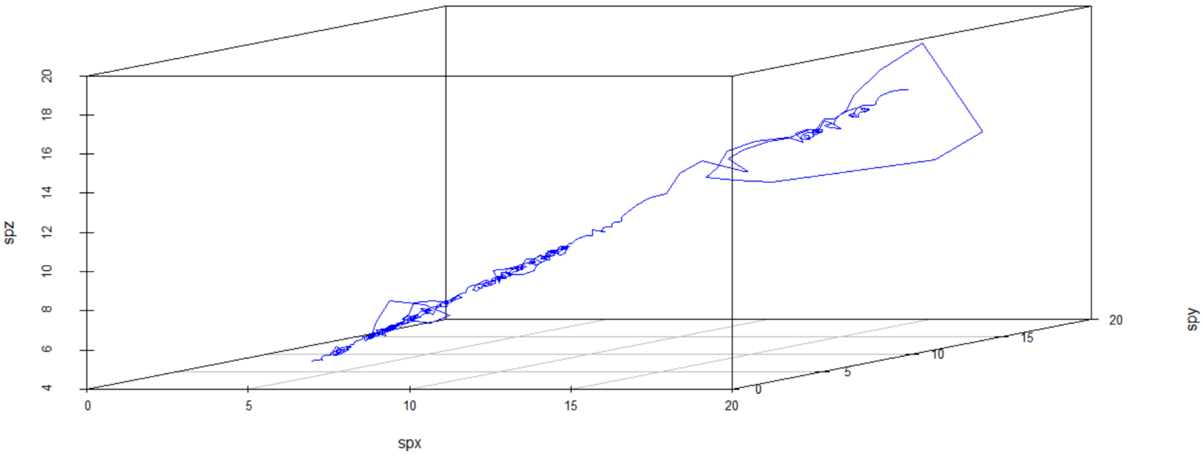

Figure 1: Phase-Space Representation of Currency Basket Time Evolution

Source: Author

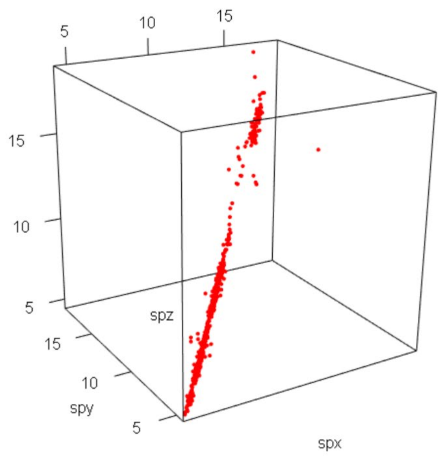

Figure 2: Multiple Equilibria in 3-Dimensional Embedding Space

Source: Author

The results of the analysis are depicted in Figure 3. Figure 3 plots the output of Eq. (6). In Figure 3, the maximal Lyapunov exponent, $\lambda_{max}(t)$ is computed from the slope of the curves according to Eq. (7). The slope is positive and is equal to 0,00896 on average. Tho τ, signifies iteration number. The negative values derive from the logarithm of the interval's distances (0,1]. The τ value(s) where the slope of the curves approximates zero is denoted by τ_{max} and signifies the last step of the scaling range, implying that the dynamic system is still predictable. For our data, τ_{max} = 12. For the iterations exceeding this value $\tau^* > \tau_{max}$, the points are no more affected by the chaotic attractor. Thus, the system investigated is said to jump through an unpredictable state. From Figure 3, it is straightforward that the data examined show chaotic behaviour associated with the positive maximal Lyapunov exponent.

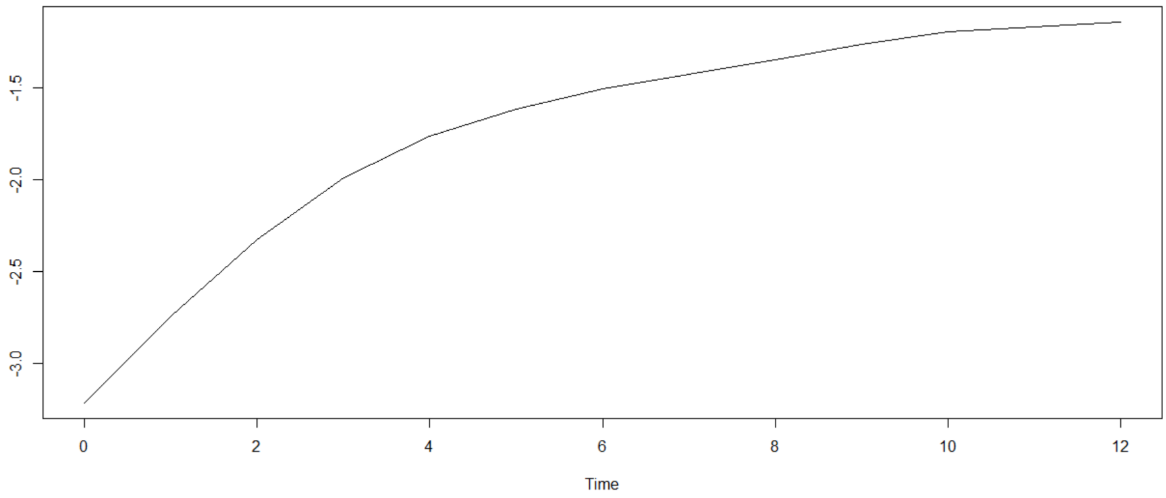

Figure 3: The time evolution of initially nearby points – average log distances, $D(\tau)$.

Source: Author

Conclusion and discussion

In this study, we investigate the Turkish Lira's chaotic behaviour against Euro and US dollar currency basket. The findings of the study show evidence of chaos in the foreign exchange markets for the period under examination. The study results show that the currency market in Turkey operates under multiple equilibria, which will lead to eventual shifts in expectations of both inflation and interest rates. In addition, the detected positive maximal Lyapunov exponent shows that complexity in foreign exchange markets has been increased, and diverging volatility eventually will generate recurrent spikes in currency value. These findings have important implications for interventions of Central banks and preventing systemic risks, as well as portfolio and risk management. Over the period from 2018 to 2022, we observe specific speculative spikes in currency values in Turkey in August 2018, November 2020, and December 2021, respectively. The country's sensitivity to short-term capital inflows-outflows and the COVID-19 pandemics crisis increased the financial vulnerability. This has augmented the need for policymakers and the Central bank to intervene in the currency markets.

It can be said that the results of this study are related to the findings of the literature in various aspects. First, the time-evolution of financial systems has long been considered from the theoretical perspective of the efficient market hypothesis introduced in Fama (1970). EMH requires certain conditions under which the market would operate efficiently. That means the market prices fully reflect every piece of information, which decision-makers have homogenously, instantaneously and rationally processed. Malkiel (2003) and Pesaran (2010) show that the market prices would be unpredictable due to this hypothesis, and financial actors cannot enjoy any arbitrage gain. Hence can obtain returns directly proportional to the risk that they have taken. In this perspective, the result of the present study challenges the Efficient market hypothesis, points out predictability and arbitrage gain in the foreign exchange market in Turkey and supports the results of Hsieh (1993), Das & Das (2007), Serletis & Dormmar (1996), Vasilios et al. (2019).

Second, the results of this study are related to the findings of the literature on the linkage between information (macroeconomic news, policy news) diffusion and price fluctuations in foreign exchange markets. Indeed, macroeconomic news drive fluctuations in both price and volatility and the impact of macroeconomic announcements is far more significant than previously envisaged (Bandi & Reno,2016, Todorov & Tauchen, 2011). The finding of our study is consistent with the reporting of these studies since detected chaotic structure increases complexity in the financial market as long as available information disrupts. Lowering available information increases the uncertainty in the financial system, leading to exchange rate volatility. This study result also sheds light on the consequences of the economic policies which focus on interventions in foreign exchange markets. The Central bank's direct foreign currency sales in currency markets may increase the magnitude of the volatility. Therefore, policymakers should better identify the conditions under which they will employ interventions in financial markets. From this perspective, it can be said that this study's results support similar studies on volatility. It has been observed in previous studies that associated volatility cannot respond to these interventions under certain conditions (Oduncu, Akcelik & Ermisoglu 2013), and hence financial stability is disrupted. Vasilios et al. (2019) recently examined the exchange rates in Brazil, Russia, India, China and South Africa (BRICS) in historical data. The authors report evidence of chaos in all currencies. Our findings are consistent with the authors' reporting in that for the most important emerging market bloc, interventions in the foreign exchange markets violate the efficiency. Our results suggest that the direct intervention will not give the desired results, conditional on the chaotic behaviour of the currency markets.

Limitations and recommendations for future research

Although this study has shown that foreign exchange markets in Turkey exhibit chaotic behaviour and diverge from efficient dynamics and outcomes, further research will be needed to investigate the micro-foundations of chaotic behaviour (Ozkaya, 2015). Moreover, future research may consider additional variables to identify local and global effects, such as Credit default swap level and the Volatility Index (VIX). On the other hand, it is considered that it would be helpful to evaluate other factors that were not evaluated in this study, such as the effect of policy makers' choices and volatility spill over in future research. Accordingly, future studies should focus on the causes of persistent volatility in emerging markets currencies, particularly in Turkish Lira, amid extraordinary monetary policies of Central banks worldwide.

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The author(s) has (have) no conflict of interest to declare.

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